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Experimental Rock Deformation - The Brittle Field

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2005, X, 348 p. 87 illus., Hardcover

ISBN: 3-540-24023-3

## The Role of Pore Fluids

### 7.1 Introduction

Almost all rocks are porous to some degree and, under natural conditions, the pores are likely to contain a fluid phase. This fluid will often be water or carbon dioxide, but other fluids such as air, gas, oil, or partial rock melt may also be involved. The presence of the fluid can affect the behaviour of the rock through both chemical and mechanical interactions. The chemical effects are particularly important for geological processes such as metasomatism and metamorphism and they also have some interaction with brittle deformation. However, in this chapter we shall concentrate mainly on the mechanical influence, where it is the pressure of the fluid that is of primary importance, with implications in both geology and engineering, such as in the behaviour of faults (Hickman, Sibson and Bruhn 1995).

The pore structure of rocks has been investigated by a variety of methods, including optical microscopy, X-ray tomography, scanning electron microscopy, and inference from physical properties (see Sect. 5.7.2). In general, a bimodal distinction can be made between pore space that is of more or less equant shape and that which is of markedly non-equant shape (Walsh and Brace 1966). These classes are often distinguished as “pores” and “cracks”, respectively, and their combination can be treated as “double porosity” (Berryman and Wang 1995). However, one can also distinguish a third class of pore space, which consists of “channels” such as those along three-grain junctions in equilibrated polycrystalline structures with dihedral angles between  $0^\circ$  and  $60^\circ$  (Smith 1948; Bulau, Waff and Tyburczy 1979; Hay and Evans 1988). The connectivity of the pore structure is of special relevance for such properties as permeability and electrical conductivity of fluid-saturated rock (Sect. 5.5) and, in this connection, it is common to distinguish between “pores” and “throats”, the latter being of predominating importance in determining the permeability or conductivity.

Many of the ideas about the mechanical role of pore fluid pressure were first developed in soil mechanics but here they are applied to rocks. Before describing the experimental findings and allied theoretical studies, it is necessary to consider briefly the conventions used in describing the stresses when pore fluid pressure is present and to introduce the effective stress notion which underlies many discussions of pore pressure effects.

## 7.2 The Notion of Effective Stress

It is normally convenient and appropriate initially, for experimental work and for applications, to treat as independent measurable stress variables the pore fluid pressure and the total macroscopic stress. The total macroscopic stress components are defined in the conventional way by considering the total forces, including those from any fluid phases, that act on the faces of an elementary cube. The dimensions of the cube are, however, chosen to be sufficiently large compared with the pore dimensions that statistical homogeneity is approximated. This continuum approach has been clearly set out by Biot (1941), Jaeger and Cook (1979, p. 211), Nur and Byerlee (1971), Rice and Cleary (1976), Detournay and Cheng (1991) and Wang (2000). The stresses thus defined are directly measurable.

There are more sophisticated theoretical approaches based on the notion of a mixture treated as a continuum, in which partial stress and deformation measures are defined for the individual components, somewhat as for the variables in chemical thermodynamics of multicomponent systems, but they need not be developed here (see, e.g., Schiffman 1970; Garg 1971; Morland 1972; Berryman 1995; Coussy 1995). An alternative theoretical approach is to try to consider the actual stresses within the solid particles or framework, as in inclusion-based models (for example, Endres 1997; Berryman 1998). However, the following discussion of experimental work will be in terms of the pore pressure and the total macroscopic stress as defined above.

The gross mechanical behaviour of a porous solid, such as in consolidation or in shearing deformation, either elastic or inelastic, depends in general both on the total macroscopic stress and on the pore pressure. In practice, however, this dependence can usually be expressed in terms of an *effective stress*. This is a notional macroscopic stress, which is derived from the measured total macroscopic stress and the pore pressure, and which is treated as the only stress variable governing the behaviour in question. That is, it is the stress that is *effective* in controlling a particular behaviour. This view of effective stress is clearly set out by Skempton (1961), Robin (1973) and Berryman (1992), and is adopted here because of its generality. There is an alternative, widely used approach in which the convention is adopted of *defining* the “effective stress” as  $\sigma_{ij} - p\delta_{ij}$  where  $\sigma_{ij}$  is the total macroscopic stress and  $p$  the pore pressure (e.g., Lambe and Whitman 1969, p. 241; Jaeger and Cook 1979, p. 219); the stress defined more narrowly in this way is better described as the *conventional effective stress* (cf. Garg and Nur 1973).

Experimental observations commonly indicate that, at least for isotropic behaviour, the effective stress can be written in the form  $\sigma_{ij} - \alpha p\delta_{ij}$  where  $\alpha$  is a constant depending on the particular property or situation. The principal effective stress components are then  $\sigma_1 - \alpha p$ ,  $\sigma_2 - \alpha p$ ,  $\sigma_3 - \alpha p$  where  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  are the principal total macroscopic stress components.

In the case of the shear strength and consolidation properties of soils, it has been well established that  $\alpha$  can generally be taken to be equal to 1 as a very close approximation (references are given below). This condition is often referred to as “Terzaghi’s principle”, or alternatively as the law of effective stress (“conventional effective stress law” in the terminology adopted here). Terzaghi’s rule represented the first introduc-

tion of the notion of effective stress in soil mechanics. It is to be noted that it was introduced on an empirical basis specifically in connection with consolidation and shear strength of soils; see Skempton (1961) for an account of the evolution of Terzaghi's ideas, with reference to his original papers, and of its further verification by Rendulic.

Terzaghi's rule that  $\alpha = 1$  also seems to have a wide empirical validity for the inelastic behaviour of rocks (see Sect. 7.4 for experimental studies). However, it should be borne in mind that it does not have a clear theoretical basis. Values of  $\alpha$  that differ from 1 have been reported in various types of porous rock and in joints (see, for example, Gangi and Carlson 1996; Boitnott and Scholz 1990; Kwon et al. 2001) although Bernabé (1987) showed that, in the case of permeability of a compact rock,  $\alpha$  tended to approach unity after cycling. For a review of values of  $\alpha$  in relation to various properties, see Sect. 7.3.1. If the solid skeleton can be considered to be a single homogeneous constituent, then the value of  $\alpha$  is predicted to be less than or equal to 1 in relation to elastic and transport properties (Nur and Byerlee 1971; Walsh 1981; Berryman 1992). Sometimes  $\alpha$  is substantially greater than 1, for example,  $\alpha = 4$  for permeability in Berea sandstone (Zoback and Byerlee 1975c). Berryman (1992) showed that if the solid skeleton of such a clayey sandstone can be considered to be made up of two or more distinct constituents, then it is plausible for  $\alpha$  to be greater than 1 if the constituent properties are significantly different. The situation may be more complicated when more than one fluid is present, as in unsaturated soils and rocks. Even when  $\alpha = 1$ , there may be advantages in decomposing the effective stress into hydrostatic and deviatoric components (Skempton 1954; Cornet and Fairhurst 1974).

### 7.3 Elastic Properties of Porous Media

#### 7.3.1 Linear Poroelasticity

Following Biot (1941), reprinted in Tolstoy (1992), linear elastic deformation of a porous body is conveniently described by

$$\varepsilon_{ij} = \frac{\sigma_{ij} - \sigma_m \delta_{ij}}{2G} + \frac{\sigma_m \delta_{ij}}{3K} - \frac{p \delta_{ij}}{3H} \quad (7.1)$$

where  $\sigma_m = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3$  (see also Jaeger and Cook 1979, pp. 211–214; Rice and Cleary 1976; Kümpel 1991; Wang 2000). The quantities  $\varepsilon_{ij}$  are the macroscopic infinitesimal strain components, determined from the displacement of points in the solid parts of the body, and  $\sigma_{ij}$  are the total macroscopic stress components. The elastic parameters  $G$  and  $K$  are the macroscopic shear and bulk moduli of the porous rock in the absence of a pore pressure or, equivalently, the moduli in a saturated sample determined under drained condition at a constant pore pressure. The additional parameter  $H$  determines the further contribution to the strain that arises when pore pressure  $p$  is introduced. Thus, the macroscopic strain tensor for the porous body is determined by three material constants,  $G$ ,  $K$ , and  $H$ , all of which can be determined by direct laboratory measurement on the porous body. The coefficient  $1/H$  is not

considered in classical elasticity theory. It represents the bulk volume change induced by a pore pressure change while maintaining the applied stress constant. The above description refers, of course, only to statistically homogeneous and isotropic bodies with fully connected porosity and adequate permeability for maintaining a drained condition with uniformity in pore pressure.

The other macroscopic quantity of interest is the change in pore fluid content,  $\zeta$ , defined to be the volume of fluid storage per unit bulk volume of the porous material (Biot 1941; Rice and Cleary 1976; Wang 2000). In response to the macroscopic stress or pore pressure, the change in fluid content is given by

$$\zeta = \frac{\sigma_m}{H} - \frac{p}{R} \quad (7.2)$$

where  $R$  is a fourth material constant which can be directly measured in the laboratory by monitoring the change in fluid storage induced by a pore pressure change while maintaining the applied stress constant. The storage change arises due to the concomitant changes in porosity  $\phi$  and fluid volume  $V_f$ . If  $K_f$  is the bulk modulus of the fluid, then the storage change is  $\zeta = \Delta\phi - \Delta V_f/V$  where  $V$  is the reference bulk volume and  $\Delta V_f = \Delta p V_f/K_f$  (Berryman 1992; Wang 2000).

The elastic behaviour of the porous solid and its fluid content are therefore determined as functions of the macroscopic stresses and the pore pressure by four material constants,  $G$ ,  $K$ ,  $H$ , and  $R$ , treating the porous material statistically as an isotropic continuum. If the porosity  $\phi$  and the bulk modulus  $K_s$  of the material forming the solid parts of the porous body as well as the bulk modulus  $K_f$  of the fluid are known, they can be related to the two poroelastic constants by:

$$\frac{1}{H} = \frac{1}{K} - \frac{1}{K_s} \quad (7.3)$$

$$\frac{1}{R} = \frac{1}{H} + \phi \left( \frac{1}{K_f} - \frac{1}{K_s} \right) = \frac{1}{K} - \frac{1+\phi}{K_s} + \frac{\phi}{K_f} \quad (7.4)$$

These relations are readily derived by considering the behaviour when the macroscopic stress is a hydrostatic pressure equal to the pore pressure  $p$  (Nur and Byerlee 1971; Wang 2000). Here we have assumed that the bulk and pore compressibilities of an unjacketed sample are equal to that of the solid material forming the skeleton of the porous body. This widely used assumption is valid if every part of the void space is continuous and accessible to fluid, the solid phase is homogeneous and isotropic, and the fluid is chemically inert with respect to the solid phase (Rice and Cleary 1976; Rudnicki 2001).

This choice of a set of four independent coefficients for characterizing the linear poroelastic behaviour is not unique. For instance, instead of the two coefficients  $H$  and  $R$  one may use two dimensionless numbers introduced by Biot and Willis (1957) and Skempton (1961). In terms of  $H$  and  $R$  the Biot-Willis coefficient is given by

$$\alpha_m = \frac{K}{H} = 1 - \frac{K}{K_s} \quad (7.5)$$

and Skempton's coefficient by

$$B = \frac{R}{H} = \frac{1/K - 1/K_s}{1/K - 1/K_s + \phi(1/K_f - 1/K_s)} \quad (7.6)$$

These two parameters are of interest in several "effective stress laws". While the Eqs. 7.1 and 7.2 express completely the macroscopic linear poroelastic behaviour, they can be written more simply in terms of effective stresses  $\sigma'_{ij} = \sigma_{ij} - \alpha p \delta_{ij}$ . The relations for the bulk strains are then given by

$$\varepsilon_{ij} = \frac{\sigma'_{ij} - \sigma'_m \delta_{ij}}{2G} + \frac{\sigma'_m \delta_{ij}}{3K} \quad (7.7a)$$

with  $\sigma'_m = (\sigma'_{11} + \sigma'_{22} + \sigma'_{33})/3$  and  $\alpha$  given by the Biot-Willis coefficient as defined in Eq. 7.5:

$$\alpha = \alpha_m = 1 - \frac{K}{K_s} \quad (7.7b)$$

This relation was derived by Biot (1957) and by Nur (1969a) and the latter supported it by experimental results for Westerly granite, thereby confirming earlier suggestions by Geertsma (1957) and Skempton (1961). In soil mechanics it is commonly observed that the bulk material is significantly more compressible than the solid grains, so that  $K_s \gg K$  and therefore  $\alpha_m$  and the effective stress parameter  $\alpha$  can both be taken to be unity.

Similarly the fluid content  $\zeta$  can be related to the mean effective stress  $\sigma'_m$  by

$$\zeta = \frac{\sigma'_m}{H} \quad (7.8a)$$

where  $\alpha$  in this case is given by the reciprocal of Skempton's coefficient  $B$  as defined in Eq. 7.6

$$\alpha = \frac{1}{B} = 1 - \phi \frac{(1/K_f - 1/K_s)}{(1/K - 1/K_s)} \quad (7.8b)$$

One can isolate that part of the fluid content change that arises from porosity change by considering an incompressible fluid (with  $K_f \rightarrow \infty$ ), which then gives the effective stress parameter  $\alpha$  for porosity change (Robin 1973; Berryman 1992)

$$\alpha = 1 - \frac{\phi}{\alpha_m} \frac{K}{K_s} = 1 - \frac{\phi}{K_s(1/K - 1/K_s)} \quad (7.8c)$$

In soil for which the solid material can be considered as incompressible ( $K_s$  infinitely large), then the  $\alpha$ 's in Eqs. 7.8a and 7.8c for bulk and pore volume changes can both be taken to be unity.

Note that the expressions for  $\alpha$  in the effective stresses differ in Eq. 7.7b for the bulk strains, in Eq. 7.8b for fluid content, and in Eq. 7.8c for the pore volume. This illustrates

the statement of Nur and Byerlee (1971) that different elastic processes can be controlled by different “effective stress laws”.

So far, only the open system, in which the pore pressure is an independent variable, has been considered. For the undrained case, the amount of pore fluid in the body is fixed (Gassmann 1951; Brown and Korringa 1975) and therefore the change in fluid content  $\zeta = 0$ . Under undrained condition, significant pore pressure excess may develop in response to compressive loading. This limiting case is of interest as a reasonable approximation for certain tectonic deformations, such as the instantaneous response of groundwater levels to the static strain field of an earthquake and the hydrologic response of a confined aquifer to earth tides (Roeloffs 1996). It can be seen from Eqs. 7.2 and 7.6 that with  $\zeta = 0$  Skempton's coefficient gives the ratio between the change in pore pressure under undrained condition and the change in mean stress, so that  $\Delta p = -B \Delta \sigma_m$ . In soil mechanics, it is commonly assumed that Skempton's pore pressure coefficient  $B = 1$  for soil saturated with water (Holtz and Kovacs 1981) since the bulk medium is much more compressible than either the water or the solid grains ( $K_s > K_f \gg K$ ). For such an unconsolidated material the storage coefficient  $1/R$  is, according to Eq. 7.4, given by  $1/R = 1/K + \phi/K_f$ , a definition that is widely adopted in hydrology (Domenico and Schwartz 1998). Fredrich, Martin and Clayton (1995) obtained values of  $B$  close to 1 at effective pressures up to 50 MPa for tuff samples with porosities of 37% and 40%. The tuff is similar to soil in that the bulk compressibility is very high.

### 7.3.2

#### Experimental Measurement of Poroelastic Properties

Complete characterization of the behaviour of an isotropic rock requires the independent measurement of at least four poroelastic constants in drained or undrained tests. There is a paucity of such measurements under controlled conditions of pore and confining pressures (Table 5). However, if two of the drained moduli ( $K$  and  $G$ ) have been measured, then the values of  $R$  and  $H$  can be inferred from Eqs. 7.3 and 7.4 if the bulk moduli  $K_s$  and  $K_f$  and the porosity  $\phi$  are known. Taking such an approach Rice and Cleary (1976) compiled and estimated poroelastic constants for 6 rocks, using the experimental data of Haimson and Fairhurst (1970), Nur and Byerlee (1971), Rummel (1974) and Zoback and Byerlee (1975c). Their compiled data are also included in Table 5.

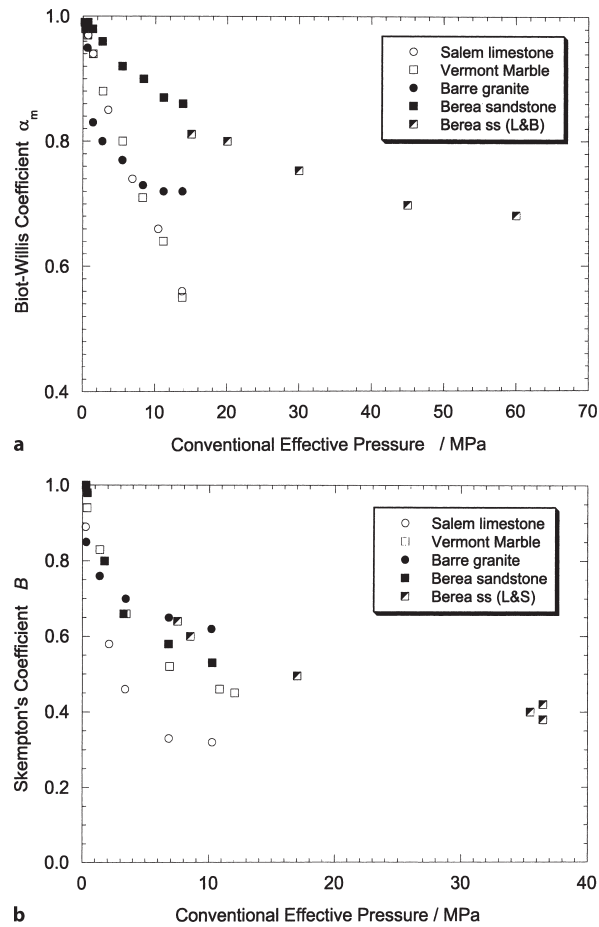
Berryman (1992) pointed out that the Biot-Willis coefficient is bounded by the porosity and unity, so that  $\phi \leq \alpha_m \leq 1$ . While the experimental data in Table 5 fall within these bounds, the coefficient  $\alpha_m$  seems not to be systematically related to the porosity  $\phi$ . This is not surprising since the bulk constants  $G$  and  $K$  themselves cannot, in general, be expressed in terms of the porosity and the elastic constants of the solid material because they also depend on the shape of the pores; (cf. Walsh 1965a,b; Warren 1973; Zimmerman 1991). In Fig. 54a we compile data on the Biot-Willis coefficient as a function of the conventional effective pressure for a granite, marble, limestone and sandstone. We calculated the  $\alpha_m$  values from the bulk moduli data of Mesri, Adachi and Ullrich (1976) using Eq. 7.5. The recent data of Lockner and Beeler (2003b) for Berea sandstone at higher effective pressures are also included. At relatively low effective

**Table 5.** Laboratory data on poroelastic constants for compact and porous rocks

| Rock                | $\phi$ | $K$ (GPa) | $K_s$ (GPa) | $\alpha_m$ | $B$  | Pressure (MPa)                | Reference                  |
|---------------------|--------|-----------|-------------|------------|------|-------------------------------|----------------------------|
| <b>Granite</b>      |        |           |             |            |      |                               |                            |
| Westerly            | 0.01   | 25        | 45          | 0.47       | 0.85 |                               | Rice and Cleary 1976       |
| Charcoal            | 0.02   | 35        | 45          | 0.27       | 0.55 |                               | Rice and Cleary 1976       |
| Barre               | 0.027  | 15        | 53          | 0.72       | 0.62 | $P_c - p \sim 10$             | Mesri et al. 1976          |
| <b>Carbonate</b>    |        |           |             |            |      |                               |                            |
| Tennessee marble    | 0.02   | 40        | 50          | 0.19       | 0.51 |                               | Rice and Cleary 1976       |
| Vermont marble      | 0.021  | 25        | 69          | 0.64       | 0.46 | $P_c - p \sim 10$             | Mesri et al. 1976          |
| Salem limestone     | 0.126  | 13        | 38          | 0.66       | 0.32 | $P_c - p \sim 10$             | Mesri et al. 1976          |
| Indiana limestone   | 0.13   | 21.2      | 72.6        | 0.71       | 0.46 | $P_c - \alpha_m p \sim 20-35$ | Hart and Wang 1995         |
| Tonnerre limestone  | 0.13   | 19.3      | 41.4        | 0.53       | 0.20 |                               | Fabre and Gustkiewicz 1997 |
| Chauvigny limestone | 0.165  | 16.3      | 52.6        | 0.69       | 0.20 | $P_c \sim 100$                | Fabre and Gustkiewicz 1997 |
| Lavoux limestone    | 0.219  | 13.8      | 58.9        | 0.77       | 0.30 | $P_c \sim 50$                 | Fabre and Gustkiewicz 1997 |
| Lixhe chalk         | 0.428  | 3.8       | 42.5        | 0.91       | 0.35 |                               | Fabre and Gustkiewicz 1997 |
| <b>Sandstone</b>    |        |           |             |            |      |                               |                            |
| Ruhr                | 0.02   | 13        | 36          | 0.65       | 0.88 |                               | Rice and Cleary 1976       |
| Weber               | 0.06   | 13        | 36          | 0.64       | 0.73 |                               | Rice and Cleary 1976       |
| Fontainebleau       | 0.06   | 30.9      | 35.2        | 0.12       | 0.25 | $P_c = 90$                    | Fabre and Gustkiewicz 1997 |
| Vosges (yellow)     | 0.17   | 17.4      | 42.5        | 0.91       | 0.46 |                               | Fabre and Gustkiewicz 1997 |
| Vosges (red)        | 0.18   | 13.9      | 38.6        | 0.64       | 0.35 |                               | Fabre and Gustkiewicz 1997 |
| Berea               | 0.19   | 8         | 36          | 0.79       | 0.62 |                               | Rice and Cleary 1976       |
| Berea               | 0.19   | 6.6       | 28.9        | 0.77       | 0.75 | $P_c - \alpha_m p = 10$       | Hart and Wang 1995         |
| Berea               | 0.203  | 4.7       | 36.3        | 0.87       | 0.53 | $P_c - p \sim 10$             | Mesri et al. 1976          |

**Fig. 54.**

**a** The Biot-Willis coefficient and **(b)** the Skempton coefficient as function of conventional effective pressure. The Berea sandstone data at elevated pressures are from Lockner and Beeler (2003b) and Lockner and Stanchits (2002). All other data are from Mesri et al. (1976)



pressures the bulk modulus  $K$  is very small due to the presence of numerous open cracks and pores, and consequently the values of  $\alpha_m$  are close to unity. At elevated pressures crack closure results in appreciable decrease of the Biot-Willis coefficient.

Skempton's coefficient  $B$  is expected to fall between 0 and 1, and the laboratory data compiled in Table 5 are within this range. The  $B$  values seem not to be systematically correlated with porosity. We compile in Fig. 54b the data of Mesri, Adachi and Ullrich (1976) on Skempton's coefficient of water-saturated samples as a function of the conventional effective stress, as well as the data of Lockner and Stanchits (2002) for Berea sandstone at higher effective pressures.  $B$  values close to 1 were measured at low effective pressures, but they fall to values between 0.33 and 0.69 at effective pressures of 10 MPa or so. Similar behaviour was observed in saturated dolomite and sandstone (Berge, Wang and Bonner 1993; Fredrich, Martin and Clayton 1995; Green and Wang 1986). The rapid decrease of  $B$  with increasing effective pressure can again be attributed to the increase of the bulk modulus  $K$  induced by crack closure.

In the case of an anisotropic porous body, the scalar coefficients  $\alpha_m$  and  $B$  are replaced by a second rank tensor (cf. Brown and Korrington 1975; Carroll 1979). Cheng (1997) presented explicit relations for these tensor coefficients. In undrained triaxial compression experiments on two sandstones, Lockner and Stanchits (2002) observed that stress-induced anisotropy can be so significant that both the Biot-Willis and Skempton's coefficients cannot be approximated as scalar quantities. Their data show that in Berea sandstone Skempton's coefficient for the undrained response to an increase in the axial stress can be as much as four times that for corresponding response to the transverse stress. Such a poroelastic response to deviatoric stresses seems to arise from the anisotropic stiffening from crack closure.

The simple theory of the elastic deformation of porous solids given above serves to illustrate in principle the approach to the inclusion of pore pressure effects in terms of effective stress. Similar considerations can be extended to the influence of pore pressure on the velocities of elastic wave propagation; see Banthia, King and Fatt (1965) and Todd and Simmons (1972) for some observations and for references to theory. However, this type of theory is limited to small strains, beyond which non-linear effects soon become important; for introduction to the non-linear aspects, see Walsh (1971a), Biot (1973), Garg and Nur (1973) and Guéguen, Dormieux and Buotéca (2004).

## 7.4 Brittle Failure – Experimental

### 7.4.1 Drained Tests

There have been many experimental studies on the influence of pore pressure on brittle fracture in rocks because of its importance in fields such as tectonics, seismology and geotechnical engineering. The experimental work has normally focused on situations in which the fluid-containing pores are interconnected, so that the pore fluid pressure can be changed or controlled by introducing or removing some fluid. This is done by connecting the specimen to an external pressure source (see Chapt. 2). Most of the work on rocks has been done with the external pressure source connected continuously to the pore system of the rock, with the aim of maintaining constancy of pore pressure; this is the arrangement known in soil mechanics as the “drained test”. The alternative arrangement (“undrained test”), in which the specimen is isolated from any external fluid systems during the test, has been used less often. The undrained condition implies that, in the absence of hydration/dehydration reactions or other internal fluid consumption/generation, the amount of fluid in the specimen is held constant during the test but the pore fluid pressure will tend to change (see further under Sect. 7.4.2).

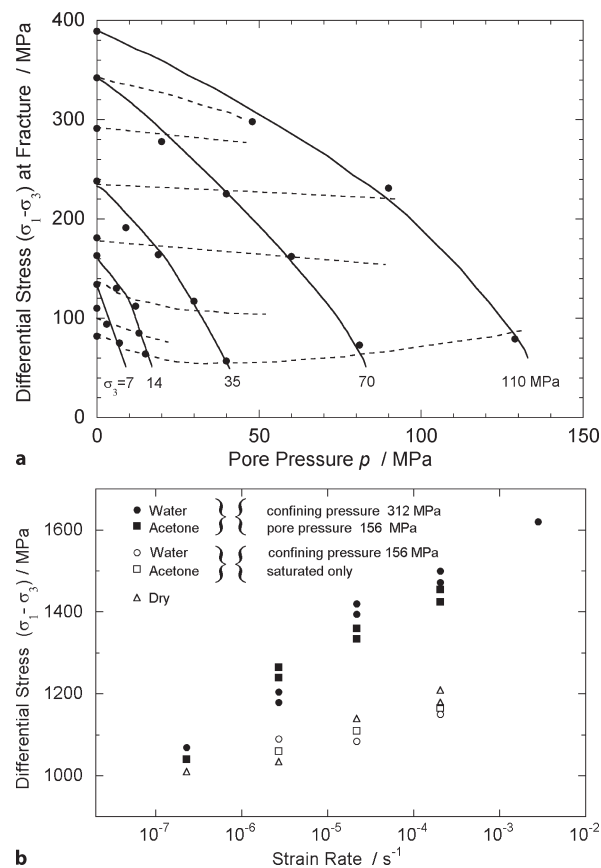
Experimental details covering such aspects as saturation of specimens, control of pore pressure, and measurement of the amount of fluid moving in or out of pores during testing have been given by Robinson (1959), Heard (1960), Handin et al. (1963), Murrell (1965), Donath (1966), Neff (1966), Lane (1970), Heck (1972) and Chiu, Johnston and Donald (1983). The pore fluid used has normally been water except when the

chemical activity of water was to be avoided or when the influence of different viscosities was to be tested. Pre-saturation of the specimen by evacuating the air while the specimen is immersed in the fluid helps to minimize complications from incomplete saturation (e.g., Neff 1966; Heck 1972; Rutter 1972b). However, there remains the frequent problem of attaining equilibrium in pore pressure through the sample when the permeability is low. Connecting the pore pressure pumping system to both ends of the specimen is sometimes done to alleviate this problem (Robinson 1959; Murrell 1965; Neff 1966; Heck 1972).

There is a considerable body of experimental work showing that to a fairly good approximation Terzaghi's effective stress rule governs the shear failure of a wide variety of rocks, including sandstone, limestone, dolomite, shale, and siltstone (Robinson 1959; Serdengecti and Boozar 1961; Handin et al. 1963; Murrell 1965; Dunn, LaFountain and Jackson 1973; Byerlee 1975; Gowd and Rummel 1977; Dropek, Johnson and Walsh 1978; Schmitt and Zoback 1989). An example is given in Fig. 55a. That is, in general, the differential stress for shear failure of a particular rock is approximately the same at the same "effective confining pressure" when the latter is taken to be the total confining

**Fig. 55.**

**a** Influence of pore pressure  $p$  on differential stress at fracture in triaxial compression tests at various confining pressures  $\sigma_3$  for a sandstone of 21% porosity, showing approximate conformity with the Terzaghi effective stress rule; the *broken lines* are lines of constant "effective pressure"  $\sigma_3 - p$  (after Murrell 1965).  
**b** Differential stress at fracture of Westerly granite as a function of strain rate at two confining pressures but at the same "effective pressure", showing departure from Terzaghi's effective stress rule at strain rates above  $10^{-6} \text{ s}^{-1}$  (after Brace and Martin 1968)



pressure minus the pore pressure ( $\alpha = 1$ ). However, apparent exceptions have been noted. Heard (1960) found that the differential stress for brittle failure in Solnhofen limestone increased with total confining pressure even though the conventional effective pressure was held constant. Robinson's (1959) results for Indiana limestone showed a similar trend, as did also the results of Handin et al. (1963) for Hasmark dolomite, a shale, and a siltstone (but only with water in the latter case, not with kerosene). Thus, it was concluded by Handin et al. (1963) that "the important mechanical properties – ultimate strength and ductility – are functions of the [conventional] *effective stresses*, provided that (a) the interstitial fluid is inert relative to the mineral constituents of the rock so that pore pressure effects are purely mechanical, (b) the permeability is sufficient to allow pervasion of the fluid and furthermore to permit the interstitial fluid to flow in or out of the rock during the deformation so that the pore pressure remains constant and uniform throughout (the test is "drained"), and (c) the rock is a sandlike aggregate with connected pore space, the configuration of which insures that the pore ("neutral") pressure is transmitted fully throughout the solid phase."

The conclusion of Handin et al. (1963) about the role of permeability has been confirmed by detailed work on low permeability rocks at slow strain rates by Brace and Martin (1968); see also Brace (1969b) and Martin (1980). They compared fracture strengths at zero pore pressure and 156 MPa confining pressure with those at 156 MPa pore pressure and 312 MPa confining pressure – the conventional effective pressure in both cases being 156 MPa. Several igneous rocks of 0.7% or less porosity and a sandstone of 2.6% porosity were tested. The porosities were measured at pressure, presumably by fluid take-up, so as to ensure comparable conditions of crack closure. Typical results for Westerly granite (porosity 0.7%) are given in Fig. 55b, showing that when the strain rate is reduced to  $10^{-7} \text{ s}^{-1}$  Terzaghi's effective stress rule is clearly obeyed; this strain rate is called the "critical strain rate". The apparent departures from Terzaghi's rule at higher strain rates were found to be independent of the chemical nature of the fluids (water and acetone) provided the viscosities were similar. The influence of viscosity is illustrated in tests on the sandstone, for which the critical strain rate is  $10^{-4} \text{ s}^{-1}$  when water is used as pore fluid and  $10^{-7} \text{ s}^{-1}$  when a silicone fluid of nearly three orders of magnitude greater viscosity is used; this relative change in critical strain rate is, therefore, consistent with the predicted relative change in the rate at which pressure equilibration with an external source could be established. The failures, single major faults with subsidiary small fractures in all cases, had the same appearance regardless of pore pressure conditions.

In a similar study of Solnhofen limestone near its brittle-ductile transition, Rutter (1972a) found a critical strain rate of  $10^{-6} \text{ s}^{-1}$  for the apparent validity of Terzaghi's effective stress rule when water is the pore fluid; the porosity, measured by saturating with water at atmospheric pressure, was 5.3%. In a study of Nevada Test Site tuff, Christensen, Green and Jones (1973) found a critical strain rate of  $10^{-3} \text{ s}^{-1}$ . It is to be noted, however, that, as emphasized by Ladanyi (1970), the behaviour above the critical strain rate in these observations need not represent a real failure of Terzaghi's effective stress rule within the specimens; it may be entirely due to the actual pore pressures being different from the fluid pressure applied and measured at the surface of the specimen, this disequilibrium being accentuated by any concurrent dilatancy (cf. Gowd and Rummel 1977).

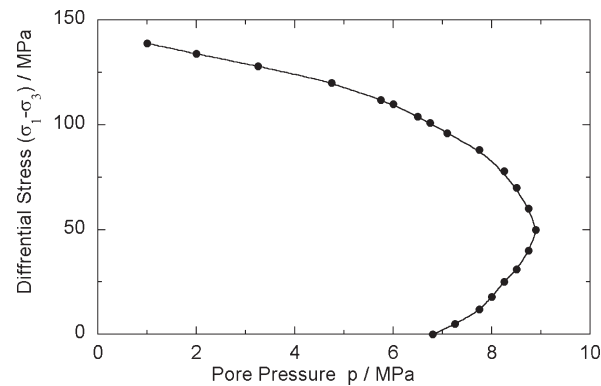
In the case of specimens without jackets to exclude the confining pressure fluid, the specimen is potentially subject to a pore fluid pressure equal to the confining pressure and the conventional effective pressure equal to zero, provided there is sufficient permeability for the penetration of the fluid to establish the pressure in the pores. Thus, in triaxial tests on specimens without jackets, brittleness tends to persist to higher confining pressures, and fracture strengths or flow stresses tend to be lower, than when jackets are used (Griggs 1936; Goguel 1948; Handin 1953; Bredthauer 1957; Heard 1960). However, only in specimens of high permeability is the strength of unjacketed specimens equal to or near that found in atmospheric pressure tests, in accord with Terzaghi's rule; the sandstone tested at 500 MPa confining pressure by Handin et al. (1963) is an example. More usually, and especially when using fairly viscous confining pressure fluids, such as kerosene at the higher pressures, the strengths of unjacketed specimens show intermediate values and tend to increase appreciably as the confining pressure increases, indicating that some effective confining pressure is being established; Solnhofen limestone (Griggs 1936) and Muddy Shale (Handin et al. 1963) are examples. Also, in such cases, the strength of unjacketed specimens is likely to appear anomalously sensitive to strain rate because of the time dependence of penetration of the fluid (Jaeger 1963).

While most of the above examples of the effects of pore fluid pressure have involved triaxial compression tests, there have also been observations confirming the approximate validity of Terzaghi's effective pressure principle in extension tests, especially in Bridgman pinching-off tests in which the conventional effective pressure is zero if the specimen is unjacketed (Jaeger 1963; Jaeger and Cook 1963; Schmitt and Zoback 1989; Bruno and Nakagawa 1991). However, this conclusion is again dependent on there being adequate permeability for establishing pore pressure equilibrium throughout the specimen (Schmitt and Zoback 1992).

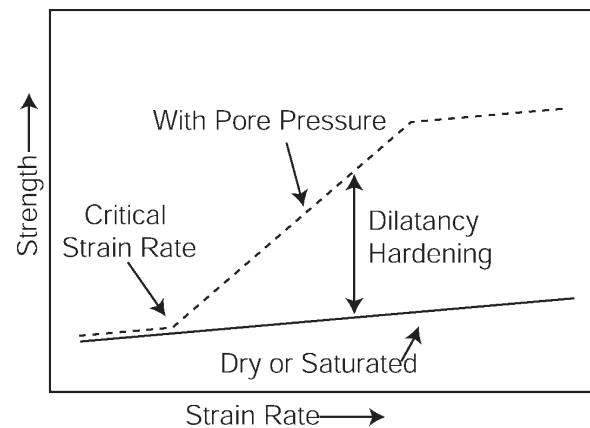
#### 7.4.2 Undrained Tests

Pore pressure effects can be substantially modified by changes in porosity during an experiment. These changes may involve dilatation or compaction, either during application of confining pressure or during deformation under differential stress (see Chapt. 6), and they are likely to be accompanied by significant changes in permeability. If the permeability is adequate and the specimen is continuously connected to the pore fluid reservoir ("drained" test), the only effect may be movement of fluid into or out of the specimen; such an effect has been widely observed (Robinson 1959; Handin et al. 1963; Rutter 1972b), and it has sometimes been monitored to give a measure of change in porosity (Read and Meredith 1989; Chiu, Johnston and Donald 1983; Zhang, Cox and Paterson 1994; Zhang, Paterson and Cox 1994; Read et al. 1995; Bernabé and Brace 1990; Wong 1990a). On the other hand, if the specimen is saturated and then isolated from the reservoir ("undrained" test), changes in pore pressure will be induced by changes in porosity accompanying deformation, as shown by Neff (1966), Aldrich (1969), Heck (1972), (also in Lane 1970), Dropek, Johnson and Walsh (1978), and Green and Wang (1986); see Fig. 56 and previous discussions in Sect. 7.3.1. Similar changes have been inferred by Ismail and Murrell (1976) to explain the behaviour in

**Fig. 56.**  
Changes in pore pressure with application of differential stress in an undrained triaxial compression test on Berea sandstone (18% porosity) at 14 MPa confining pressure, 7 MPa initial pore pressure (after Heck 1972)



**Fig. 57.**  
The concepts of dilatancy hardening and critical strain rate associated with it, deriving from observations of the type shown in Fig. 56 (after Brace and Martin 1968)



undrained tests of rocks of various porosities when water was added in excess of that needed for initial saturation.

Induced changes in pore pressure similar to those in “undrained” tests can also be expected in nominally “drained” tests when the permeability is inadequate for the maintenance of equilibrium between the pressure in the pores and in the reservoir, that is, when the strain rate is above the critical strain rate defined by Brace and Martin (1968) and mentioned in Sect. 7.4.1. An increase in porosity can then lead to a decrease in the actual pore pressure within the specimen, so that greater differential stress is needed for deformation than when constancy of actual pore pressure can be maintained. This effect, known as “*dilatancy hardening*”, has been discussed by Brace and Martin (1968) and depicted by them as in Fig. 57; see also Rutter (1972a), Ismail and Murrell (1976), Chiu, Johnston and Donald (1983), and Schmitt and Zoback (1992). The effect is similar to the effect in granular media discussed by Reynolds (1885; 1887) and Mead (1925), and it can be expected to affect sliding on fault surfaces or cataclastic deformation whenever permeability is low in relation to the strain rate and dilatation accompanies the deformation. Its relevance to seismicity has been discussed by Frank (1965, 1966) and it is an important aspect of the models for the mechanism of earth-

quakes of Nur (1972), and Scholz, Sykes and Aggarwal (1973), which have figured in discussions on earthquake predictions (for example, Mogi 1977; Sibson 1981; Crampin, Evans and Atkinson 1984).

### 7.4.3

#### Chemical Effects

An active role of the interstitial fluid through chemical or other processes independent of the applied pore pressure has often been observed. The apparent exception to Terzaghi's rule for siltstone with water as pore fluid, observed by Handin et al. (1963) and noted above, was attributed to the swelling of montmorillonite clays, which greatly reduces the permeability. However, it has been widely observed, especially in quartz-rich rocks but also in limestone, that water, including even that adsorbed from a humid atmosphere, has a specific weakening effect relative to behaviour after careful drying (Hirschwald 1912, pp. 192–196; Jaeger 1943; Terzaghi 1945; Price 1960; Serdengecti and Boozer 1961; Colback and Wiid 1965; Murrell 1965; Aldrich 1969; Podnieks, Chamberlain and Thill 1972; Rutter 1972b; Christensen, Green and Jones 1973; McCarter and Willson 1973; Parate 1973; Atkins and Peng 1974; van Eeckhout 1976; Seto et al. 1997). Alkaline solutions have also been observed to lower the fracture stress of both quartzite and quartz crystals (Griggs and Bell 1938; Fairbairn 1950), and aluminium and ferric iron-bearing solutions have a similar or even more marked effect which can be rationalized in terms of zeta-potentials (Swolfs 1972). Such observations call to mind the Rehbinder effect (Rehbinder, Schreiner and Zhigach 1948). Thus, Boozer, Hiller and Serdengecti (1963) observed weakening effects with a number of surface-active liquids on both sandstone and limestone, independently of any pore pressure effect; for example, oleic acid and water weakened both rocks, whereas *n*-hexadecane produced no effect. Robinson (1967) confirmed and extended the observations of Boozer et al. but pointed out that the additional weakening by solutions or organic chemicals over that due to water alone applied only to the yielding, not to brittle strength. Colback and Wiid (1965) also reported observations on uniaxial strength of quartzitic sandstone in a number of liquids, which show a correlation between the degree of weakening and the surface tension of the liquid (see also Vutukuri 1974, for limestone). However, as seen from the relative effects with oleic acid and water, such a correlation does not apply in the results of Boozer, Hiller and Serdengecti (1963), who make the more likely suggestion that it is the surface energy of the mineral when wetted with the fluid that is important (see also van Eeckhout 1976; Baud, Zhu and Wong 2000). On the other hand, studies of the influence of water on crack propagation in quartz (Charles 1959; Scholz 1968c; Martin 1972) suggest a specific chemical role for water in silicate rocks in assisting to break Si-O bonds (see also Swolfs 1972). These effects can be viewed as examples of crack propagation controlled by stress corrosion; see reviews by Anderson and Grew (1977), Atkinson and Meredith (1987b) and Costin (1987).

The effects of adsorbed moisture can also be treated from a point of view commonly adopted in soil mechanics for unsaturated soils. In clay soils, there are important variations in strength with variations in moisture content when no free water is present. Relative to the saturated condition, reduction in water content gives a strengthening which has been variously dealt with in terms of notions of suction, soil mois-

ture tension, pressure deficiency, negative pore pressure, etc., introduced into effective stress theory; see, e.g., papers in British National Society (1961) or Lambe (1969). This approach has not been applied much in rock mechanics but it could be relevant in fine grained rocks, especially in argillaceous rocks. Thus, Chenevert (1970) treated the strength of argillaceous shales in terms of an adsorptive pore pressure, defined as  $(RT/v) \ln(p/p_0)$ , where  $p$  and  $p_0$  are the vapour pressures of water in equilibrium with the rock and with free water, respectively,  $v$  is the specific volume of water,  $R$  is the gas constant and  $T$  the absolute temperature; the relative humidity of the atmosphere in equilibrium with the rock is taken as the measure of  $p/p_0$  (see also Schmitt, Forsans and Santarelli 1994). The adsorptive pore pressure, which is negative, that is, of the same sign as tensile stress, is then used in the same way as the pore pressure in a saturated rock to derive conventional effective stresses (for relation to swelling in shales, see Huang, Aughenbaugh and Rockaway 1986). Chenevert found that the compressibility, swelling and strength of several shales exposed to atmospheres of various relative humidities behaved as if controlled by effective stresses obtained in this way, and the same correlation was shown to apply to the quartzitic shale and quartzitic sandstone of Colback and Wiid (1965), mentioned above. The strength of granular materials containing amounts of water below the level of saturation is also of concern in the technology of such materials and has been discussed in terms of “capillary bonds” between grains (cf. Carr 1967; Frank 1971, 1972, and other references given by them).

## 7.5

### Brittle Failure – Theory of Pore Pressure Effects

The theory of the influence of pore pressure on brittle failure has encountered a number of difficulties. In particular, discussion of the theoretical basis of the dominant role of the conventional effective stress has been marked by a good deal of controversy and some confusion. Fundamentally, the theoretical problem is to relate the stresses and attendant failure phenomena on the microscopic scale (scale of the pores) to the macroscopic stress state and the macroscopic failure.

In theoretical discussion, it is important to keep clearly in mind the various ways in which stresses are defined (cf. Sect. 7.2). Firstly, there are the total macroscopic stresses, involving the totality of forces, in both solid and fluid parts, acting across a hypothetical macroscopically plane surface, the area of which is taken to include both the solid material and the pore space intersected by the surface. Secondly, there are local stresses, based on the forces acting on local parts of this hypothetical surface lying entirely within a uniform region (either solid material or pore space). Finally, there are quantities which can be described as partial macroscopic stresses, defined by considering the forces acting on only those parts of the surface that intersect either solid or fluid but dividing the measure of the forces by the total area of the macroscopic surface.

It is also important to consider brittle failure specifically since it is already evident from the previous sections on elastic deformation and on the experimental study of brittle failure that there is no simple universal definition of “effective stress” which permits a uniform treatment of all phenomena affected by pore fluid pressure. Thus, the discussion of so-called buoyancy forces and their role in sliding friction on sur-

faces between separate bodies, which dominated much of the paper and subsequent discussion of Hubbert and Ruby (1959), is not necessarily directly relevant in understanding the role of the conventional effective stress in brittle failure of initially intact rock. It should also be noted (cf. Moore 1961) that mathematically the particular decomposition of the total stress into a “neutral stress”  $p$  and a conventional effective stress has no special uniqueness. Emphasis has sometimes been put on the “neutral” character of the pore pressure, which is said to give a uniform hydrostatic reference state through the whole of the material if the pressure is transmitted to all the pores, but this concept is useful only as long as either infinitesimal elastic deformation alone is involved or elastic deformation is entirely neglected.

A common approach to rationalizing the apparent role of the conventional effective stresses in brittle failure has been to consider some sort of average value of the local stresses in the solid parts of the specimen. Considering the stress components on a potential plane of failure, it is readily shown (e.g., Skempton 1961; Jaeger and Cook 1979, p. 222) that the total normal stress component  $\sigma$  can be written as

$$\sigma = p + (1 - f)(\sigma_s - p) \quad (7.9)$$

where  $\sigma_s$  is the average value of the local normal stress in the solid parts intersected by the macroscopic element of area upon which the stress  $\sigma$  is calculated,  $f$  is the fraction of this area occupied by the pore space, and  $p$  is the pore pressure. The quantity  $f$  is generally taken to be equal to the porosity and is often called the “boundary porosity” (e.g., Robinson and Holland 1970). Equation 7.9 can be re-written as

$$(1 - f)\sigma_s = \sigma - fp \quad (7.10)$$

The quantity  $(1 - f)\sigma_s$  is equal to the total normal force transmitted through the solid parts divided by the total area; that is, it is a partial stress. By a similar consideration to that leading to Eq. 7.9, the total shear stress  $\tau$  can be written as

$$(1 - f)\tau_s = \tau \quad (7.11)$$

where  $\tau_s$  is the average value of the local shear stress in the solid parts intersected.

If it is now assumed that shear failure will occur on the surface under consideration when the average stresses in the solid parts satisfy a Coulomb condition similar to that applying to the same material with no pores, then

$$\tau_s = \tau_0 + \sigma_s \tan \varphi$$

where  $\tau_0$  and  $\tan \varphi$  are the cohesion and coefficient of internal friction of the solid material itself, or closely related parameters. Substituting from Eqs. 7.10 and 7.11 gives

$$\tau = (1 - f)\tau_0 + (\sigma - fp) \tan \varphi \quad (7.12)$$

Therefore,  $\sigma - fp$  appears in the Coulomb condition for the porous specimen instead of  $\sigma$ , that is, the “effective” shear stress is still  $\tau$ . This or similar arguments (Terzaghi

1945; Skempton 1961) have thus led to the prediction that brittle failure should be governed by effective stresses  $\sigma'_{ij} = \sigma_{ij} - \alpha p \delta_{ij}$  where  $\alpha = f$ , the “boundary porosity”.

However, the observed behaviour generally corresponds to  $\alpha = 1$ , even in rocks of low porosity, so that if the porosity is taken as the measure of  $f$  the above prediction is not obeyed (see also Robin 1973). To reconcile this discrepancy, it has become customary in soil mechanics to view the potential failure surface as a wavy surface following only in an approximate way the plane surface considered in the definition of the stresses in the above argument (e.g., Lambe and Whitman 1969, p. 242). It is then said that, since such a wavy surface can lie almost entirely within pore space by going around the grains, its boundary porosity  $f$  is close to unity. A similar view can be applied to rock but it leads to the conclusion that “the voids must consist of very narrow but continuous slits” (Terzaghi 1945, p. 786).

Such an explanation of how the conventional effective stresses ( $\alpha = 1$ ) apply to brittle failure is not very satisfying for rocks. Its deficiency probably lies mainly in treating the stresses within the solid parts as uniform, that is, in taking into account only the average value of the local stresses, and in neglecting to consider the local and propagating nature of a brittle failure. The latter point has been raised by Laubscher (1960) who suggests that during the approach to failure there is a changing microscopic geometry involving the development of a series of local fractures, so that the effective boundary porosity of a potential fracture surface increases during this development and approaches unity just before the macroscopic movement on the fracture surface begins. The analysis of such a situation is most simply approached through the Griffith model for brittle fracture.

The Griffith model has been described in Chapt. 4. It takes into account the actual stress state around an ellipsoid-shaped crack, assuming elastic behaviour prior to the local failure whereby the crack is extended, and it postulates that the crack will extend when the extra work of creating new crack surface can be provided from elastic strain energy available. If the crack contains a fluid under pressure, the theory shows that the failure condition remains of the same form provided conventional effective stresses  $\sigma_{ij} - p \delta_{ij}$  are used instead of the total stresses  $\sigma_{ij}$ , this applies in both the two-dimensional model (Murrell 1964; Jaeger 1969, p. 169; Jaeger and Cook 1979, p. 279) and the three-dimensional (Murrell and Digby 1970). Thus, Griffith theory *predicts*  $\alpha = 1$ , and, insofar as it is applicable, gives justification for the use of the conventional effective stresses in predicting brittle failure. Of course, as discussed in Chapt. 4, Griffith theory is only strictly relevant to the initiation of fracture, but similar considerations presumably apply at each stage of the propagation of the fracture and it seems plausible that the macroscopic failure of a brittle porous material with pore pressure should similarly be governed by the conventional effective stresses ( $\alpha = 1$ ).

Indeed this is the approach usually adopted to incorporate pore pressure effects into the various micromechanical models of brittle failure discussed in Chapt. 6. Underlying the fracture mechanics treatment is the usual presumption that the pore fluid is always in hydrostatic equilibrium. However, in a crack growing sufficiently rapidly, viscous drag may prevent continuous fluid penetration to the crack tip, a situation discussed by Abé, Mura, and Keer (1976) in the context of hydraulic fracturing, which represents another aspect of the critical strain rate (Sect. 7.4.1). Atkinson and Cook (1993) have modelled such a situation microdynamically.