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# CHAPITRE 1

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## Notions de base

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### 1. Calcul littéral

#### 1.1 Rappels de cours

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**Développer**, c'est transformer un produit en une somme.

**Factoriser**, c'est transformer une somme en un produit.

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$$k(a + b) = ka + kb$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a - b)(a + b) = a^2 - b^2$$

$$k(a + b) = ka + kb$$

$$-(a + b) = -a - b$$

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Exemple 1.

*Développer* :  $-3x(4 - 5x)$  ;  $(3x + 6)^2$  ;  $(5x - 1)(1 - 3x)$

- $-3x(4 - 5x) = -12x + 15x^2$
- $(3x + 6)^2 = 9x^2 + 36x + 36$
- $(5x - 1)(1 - 3x) = 5x - 15x^2 - 1 + 3x = -15x^2 + 8x - 1$

Exemple 2.

*Factoriser* :  $3x^2 + 21x$  ;  $36x^2 - 100$  ;  $25x^2 + 30x + 9$  ;  $16x - 4$

- $3x^2 + 21x = 3 \times x \times x + 7 \times 3 \times x = 3 \times (x + 7)$
- $36x^2 - 100 = (6x)^2 - 10^2 = (6x - 10)(6x + 10)$
- $25x^2 + 30x + 9 = (5x)^2 + 2 \times 5x \times 3 + 3^2 = (5x + 3)^2$
- $16x - 4 = 4 \times 4x - 4 \times 1 = 4(4x - 1)$

## 1.2 Exercices

### 1. Développer, réduire :

$$\begin{array}{lll}
 -5(2-x); & (2x+1)(x^2-3x+2); & (a+b)^3; \\
 (7x+2y)(-3); & (9x-6)(3+2x); & (a-b)^3; \\
 3x-(2-5x); & (1+x+y)^2; & (a-b)(a^2+ab+b^2). \\
 (3-4x)^2; & (11-2x)(11+2x); & 
 \end{array}$$

### 2. Factoriser :

$$\begin{array}{lll}
 101^2 - 99^2; & 5xy - 3x^2y^3; & 4 + 20x + 25x^2; \\
 7x - 14y + 28; & 9x^2 - 6x + 1; & 8xy - 32xy^2. \\
 2x^2 - 49; & & 
 \end{array}$$

## 1.3 Corrections

### 1.

$$-5(2-x) = -5 \times 2 + (-5) \times (-x) = -10 + 5x$$

$$(7x+2y)(-3) = 7x \times (-3) + 2y \times (-3) = -21x - 6y$$

$$3x - (2 - 5x) = 3x - (+2 - 5x) = 3x - 2 + 5x = 8x - 2$$

$$(3 - 4x)^2 = 3^2 - 2 \times 3 \times 4x + (4x)^2 = 9 - 24x + 16x^2$$

$$\begin{aligned}
 & (2x+1)(x^2-3x+2) \\
 &= 2x \times x^2 + 2x \times (-3x) + 2x \times 2 + 1 \times x^2 + 1 \times (-3x) + 1 \times 2 \\
 &= 2x^3 - 6x^2 + 4x + x^2 - 3x + 2 = 2x^3 - 5x^2 + x + 2
 \end{aligned}$$

$$\begin{aligned}
 & (9x-6)(3+2x) \\
 &= 9x \times 3 + 9x \times 2x - 6 \times 3 - 6 \times 2x = 18x^2 + 15x - 18
 \end{aligned}$$

$$\begin{aligned}
 & (1+x+y)^2 \\
 &= (1+x+y)(1+x+y) \\
 &= 1 + x + y + x + x^2 + xy + y + yx + y^2 \\
 &= x^2 + y^2 + 2x + 2y + 2xy + 1
 \end{aligned}$$

$$(11-2x)(11+2x) = 11^2 - (2x)^2 = 121 - 4x^2$$

$$\begin{aligned}
 & (a+b)^3 = (a+b)^2 \times (a+b) = (a^2 + 2ab + b^2)(a+b) \\
 &= a^3 + a^2b + 2a^2b + 2ab^2 + b^2a + b^3 = a^3 + 3a^2b + 3ab^2 + b^3
 \end{aligned}$$

$$\begin{aligned}
 (a-b)^3 &= (a-b)^2 \times (a-b) = (a^2 - 2ab + b^2)(a-b) \\
 &= a^3 - a^2b - 2a^2b + 2ab^2 + b^2a - b^3 \\
 &= a^3 - 3a^2b + 3ab^2 - b^3
 \end{aligned}$$

$$(a-b)(a^2 + ab + b^2) = a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3 = a^3 - b^3$$

2.

$$101^2 - 99^2 = (101 - 99)(101 + 99) = 2 \times 200 = 400$$

$$7x - 14y + 28 = 7 \times x - 2 \times 7 \times y + 7 \times 4 = 7 \times (x - 2y + 4)$$

$$\begin{aligned}
 2x^2 - 49 &= (\sqrt{2} \times x)^2 - 7^2 = (x\sqrt{2} - 7) \times (x\sqrt{2} + 7) \\
 &= (\sqrt{2} \times x)^2 - 7^2 = (x\sqrt{2} - 7) \times (x\sqrt{2} + 7)
 \end{aligned}$$

$$\begin{aligned}
 5xy - 3x^2y^3 &= xy \times 5 - 3x \times x \times y \times y \times y \\
 &= xy \times 5 - 3xy \times xy^2 = xy \times (5 - 3xy^2)
 \end{aligned}$$

$$9x^2 - 6x + 1 = (3x)^2 - 2 \times 3x \times 1 + 1^2 = (3x - 1)^2$$

$$4 + 20x + 25x^2 = (5x + 2)^2$$

$$8xy - 32xy^2 = 8xy \times 1 - 4 \times 8xy \times y = 8xy \times (1 - 4y)$$

## 2. Les puissances

### 2.1 Rappel de cours

$$a^n = \underbrace{a \times a \times \dots \times a}_n$$

$n$  facteurs

$$a \in \mathbb{R} \text{ et } n \in \mathbb{Z}$$

Règles de calcul	Exemples
$a^0 = 1$	$\left(\frac{1}{5}\right)^0 = 1$
$a^1 = a$	$\sqrt{6}^1 = \sqrt{6}$
$a^{-1} = \frac{1}{a}$	$3^{-1} = \frac{1}{3}$
$a^{-n} = \frac{1}{a^n}$	$2^{-5} = \frac{1}{2^5}$
$a^n \times a^m = a^{n+m}$	$5^3 \times 5^{-9} = 5^{-6}$
$\frac{a^n}{a^m} = a^{n-m}$	$\frac{4^8}{4^{-11}} = 4^{8-(-11)} = 4^{19}$
$(a^n)^m = a^{n \times m}$	$((-6)^2)^5 = (-6)^{10}$
$a^n b^n = (a \times b)^n$	$7^2 \times 3^2 = 21^2$
$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$\frac{4^6}{3^6} = \left(\frac{4}{3}\right)^6$

## 2.2 Exercices

### 1. Calculer :

$$5^{-3}; \quad 10^4; \quad 7^{-2}; \quad 6^{-1}$$

### 2. Écrire sous forme de puissance :

$$27; \quad 0,125; \quad \frac{4}{25}; \quad \frac{8}{216}$$

### 3. Calculer à l'aide des règles sur les puissances :

$$3^8 \times 3^{-6}; \quad 4^3 \times 4^{-1} \times 4^2 \times 4^{-4}; \quad \frac{5^8}{5^{-2}};$$

$$\frac{7^2 \times 7^{-6}}{7^9}; \quad \frac{4^2(4^6)^2}{4^{-5}4^3}; \quad \frac{10^2 \times 10^{-3} \times 10^5}{10^{-8} \times 10^6 \times 10^{-7}}$$

### 4. Simplifier :

$$\frac{x^3 x^{-9}}{x^2 x^4}; \quad \frac{a^{-8} b^7}{a^2 b^5}; \quad \left(\frac{a}{b}\right)^3 \times \frac{b^2}{a^{-1}}$$

## 2.3 Corrections

1.

$$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

$$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

$$10^4 = 10\,000$$

$$6^{-1} = \frac{1}{6}$$

2.

$$27 = 9 \times 3 = 3 \times 3 \times 3 = 3^3$$

$$\frac{4}{25} = \frac{2^2}{5^2} = \left(\frac{2}{5}\right)^2$$

$$0,125 = \frac{1}{8} = \frac{1}{2 \times 2 \times 2} = \frac{1}{2^3}$$

$$\frac{8}{216} = \frac{8}{27 \times 8} = \frac{1}{27} = \frac{1}{3^3}$$

3.

$$\begin{aligned} 3^8 \times 3^{-6} &= 3^{8+(-6)} \\ &= 3^2 = 9 \end{aligned}$$

$$\frac{4^2(4^6)^2}{4^{-5}4^3} = \frac{4^2 \times 4^{6 \times 2}}{4^{-5+3}}$$

$$\begin{aligned} 4^3 \times 4^{-1} \times 4^2 \times 4^{-4} \\ &= 4^{3-1+2-4} = 4^0 = 1 \end{aligned}$$

$$= \frac{4^{2+12}}{4^{-2}} = 4^{14-(-2)} = 4^{16}$$

$$\frac{5^8}{5^{-2}} = 5^{8-(-2)} = 5^{10}$$

$$\frac{10^2 \times 10^{-3} \times 10^5}{10^{-8} \times 10^6 \times 10^{-7}}$$

$$\frac{7^2 \times 7^{-6}}{7^9} = 7^{2+(-6)-9} = 7^{-13}$$

$$= 10^{2-3+5+8-6+7} = 10^{13}$$

4.

$$\frac{x^3 x^{-9}}{x^2 x^4} = x^{3-9-2-4}$$

$$= x^{-12}$$

$$\frac{a^{-8} b^7}{a^2 b^5} = a^{-8-2} \times b^{7-5}$$

$$= a^{-10} \times b^2$$

$$\left(\frac{a}{b}\right)^3 \times \frac{b^2}{a^{-1}}$$

$$= \frac{a^3}{b^3} \times \frac{b^2}{a^{-1}}$$

$$= a^{3+1} \times b^{2-3} = a^4 \times b^{-1}$$

### 3. Les fractions

#### 3.1 Rappel de cours

*Exemples*

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$	$\frac{2}{3} - \frac{7}{3} = \frac{2-7}{3} = -\frac{5}{3}$
$\frac{a}{c} + \frac{b}{d} = \frac{a \times d + b \times c}{cd}$	$-\frac{5}{4} + \frac{3}{7} = -\frac{23}{28}$
$a \times \frac{b}{c} = \frac{a \times b}{c}$	$6 \times \frac{7}{11} = \frac{6 \times 7}{11} = \frac{42}{11}$
$\frac{a}{c} \times \frac{b}{d} = \frac{a \times b}{c \times d}$	$\frac{3}{8} \times \frac{5}{7} = \frac{3 \times 5}{8 \times 7} = \frac{15}{56}$
$\frac{\frac{a}{c}}{\frac{b}{d}} = \frac{a}{c} \times \frac{d}{b} = \frac{a \times d}{c \times b}$	$\frac{\frac{5}{2}}{\frac{3}{7}} = \frac{5}{2} \times \frac{7}{3} = \frac{35}{6}$
$\frac{a \times c}{b \times c} = \frac{a}{b}$	$\frac{15 \times 8}{7 \times 8} = \frac{15}{7}$
$\frac{-a}{-b} = \frac{a}{b}$	$\frac{-5}{-2} = \frac{5}{2}$
$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$	$-\frac{4}{9} = \frac{-4}{9} = \frac{4}{-9}$

#### 3.2 Exercices

1. Calculer et simplifier les expressions suivantes :

$$\frac{5}{9} + \frac{7}{9} - \frac{4}{9} ; \frac{5}{13} - \frac{3}{26} ; 4 + \frac{2}{7} ; \frac{1}{5} - \frac{8}{15} + \frac{3}{45} ; \frac{5}{7} \times \frac{14}{25} ; \frac{\frac{3}{2}}{\frac{5}{3}} \times \frac{4}{5} ; \frac{1 + \frac{2}{3}}{\frac{3}{4} - \frac{5}{8}}$$

$$\frac{7x^2yz^6}{14x^4y^3z^2} ; \frac{x}{4} - \frac{12x}{5} + \frac{3x}{20} ; \frac{x^2 + 2x}{x^3} ; \frac{5y + xy}{y^2}$$

2. Simplifier les fractions après décomposition :

$$\frac{168}{63} ; \frac{143}{104} ; \frac{125}{45} ; \frac{567}{252}$$

3. Simplifier :

$$\frac{2}{x} - \frac{5}{x+3} ; \frac{x}{x+1} + \frac{2x}{x-1} ; \frac{\frac{1}{x} - 2}{2x + \frac{5}{x+1}} ; \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{2}{x}} ; \frac{\frac{x}{x^2-1}}{\frac{3}{x-1}}$$



## 3.3 Corrections

1.

$$\frac{5}{9} + \frac{7}{9} - \frac{4}{9} = \frac{5+7-4}{9} = \frac{8}{9}$$

$$\frac{5}{13} - \frac{3}{26} = \frac{5 \times 2}{13 \times 2} - \frac{3}{26} = \frac{7}{26}$$

$$4 + \frac{2}{7} = \frac{4 \times 7}{1 \times 7} + \frac{2}{7} = \frac{30}{7}$$

$$\frac{1}{5} - \frac{8}{15} + \frac{3}{45} = \frac{1 \times 9}{5 \times 9} - \frac{8 \times 3}{15 \times 3} + \frac{3}{45} = -\frac{12}{45} = -\frac{4}{15}$$

$$\frac{5}{7} \times \frac{14}{25} = \frac{5 \times 14}{7 \times 25} = \frac{5 \times 2 \times 7}{7 \times 5 \times 5} = \frac{2}{5}$$

$$\frac{\frac{3}{2}}{\frac{5}{3}} \times \frac{4}{5} = \frac{3}{2} \times \frac{3}{5} \times \frac{4}{5} = \frac{36}{50} = \frac{18}{25}$$

$$\frac{1 + \frac{2}{3}}{\frac{3}{4} - \frac{5}{8}} = \frac{\frac{5}{3}}{\frac{1}{8}} = \frac{5}{3} \times \frac{8}{1} = \frac{40}{3}$$

$$\frac{7x^2yz^6}{14x^4y^3z^2} = \frac{7x^2yz^4 \times z^2}{7 \times 2 \times x^2 \times x^2 \times y \times y^2 \times z^2} = \frac{z^4}{2x^2y^2}$$

$$\frac{x}{4} - \frac{12x}{5} + \frac{3x}{20} = \frac{5x}{20} - \frac{48x}{20} + \frac{3x}{20} = -\frac{40x}{20} = -2x$$

$$\frac{x^2 + 2x}{x^3} = \frac{x \times x + 2 \times x}{x \times x \times x} = \frac{x+2}{x^2}$$

$$\frac{5y + xy}{y^2} = \frac{5 \times y + x \times y}{y \times y} = \frac{5+x}{y}$$

2.

$$\frac{168}{63} = \frac{8 \times 7 \times 3}{3 \times 3 \times 7} = \frac{8}{3}$$

$$\frac{125}{45} = \frac{25 \times 5}{9 \times 5} = \frac{25}{9}$$

$$\frac{143}{104} = \frac{11 \times 13}{8 \times 13} = \frac{11}{8}$$

$$\frac{567}{252} = \frac{9 \times 63}{9 \times 28} = \frac{63}{28} = \frac{9 \times 7}{4 \times 7} = \frac{9}{4}$$

3.

$$\frac{2}{x} - \frac{5}{x+3} = \frac{2(x+3)}{x(x+3)} - \frac{5x}{x(x+3)} = \frac{2x+6-5x}{x(x+3)} = \frac{-3x+6}{x(x+3)}$$

$$\frac{x}{x+1} + \frac{2x}{x-1} = \frac{x(x-1)}{(x+1)(x-1)} + \frac{2x(x+1)}{(x-1)(x+1)} = \frac{x^2-x+2x^2+2x}{x^2-1} = \frac{3x^2+x}{x^2-1}$$

$$\frac{\frac{1}{x} - 2}{2x + \frac{5}{x+1}} = \frac{\frac{1-2x}{x}}{\frac{2x(x+1)+5}{x+1}} = \frac{1-2x}{x} \times \frac{x+1}{2x^2+2x+5} = \frac{(1-2x)(x+1)}{x(2x^2+2x+5)}$$

$$\frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{2}{x}} = \frac{\frac{x-(x+1)}{x(x+1)}}{\frac{2}{x}} = \frac{x-(x+1)}{x(x+1)} \times \frac{x}{2} = \frac{(x-x-1) \times x}{2x(x+1)} = -\frac{x}{2x(x+1)}$$

$$\frac{\frac{x}{x^2-1}}{\frac{3}{x-1}} = \frac{x}{x^2-1} \times \frac{x-1}{3} = \frac{x(x-1)}{3(x-1)(x+1)} = \frac{x}{3(x+1)}$$

## 4. Racines

### 4.1 Rappel de cours

La racine carrée d'un nombre positif  $a$  est l'unique nombre, noté  $\sqrt{a}$ , tel que  $(\sqrt{a})^2 = a = a^{\frac{1}{2}}$ .

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b} ; \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

*Remarque :*  $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$

$$Ex: \sqrt{4} + \sqrt{9} = 2 + 3 = 5 \neq \sqrt{4+9} = \sqrt{13}$$

Résolution de  $x^2 = a$  :

Si  $a < 0$ , il n'y a pas de solution.

Si  $a = 0$ , il y a une solution,  $x = 0$ .

Si  $a > 0$ , il y a deux solutions :  $\sqrt{a}$  ou  $-\sqrt{a}$ .

Exemple 1.

$$x^2 = 144 \Leftrightarrow x = \pm 12$$

On appelle racine  $n^{\text{ième}}$  d'un nombre  $a$  positif le nombre positif noté  $a^{\frac{1}{n}}$  tel que  $(a^{\frac{1}{n}})^n = a ; n \neq 0$

Exemple 2.

$$\sqrt[3]{64} = 4 \text{ car } 4^3 = 64 ;$$

Notation :  $a^{\frac{1}{n}} = \sqrt[n]{a}$